With one vector element now known, the sign of another vector element is determined by Eq. (8) as

$$\text{sgn}\ (q_j) = \text{sgn}\ (q_i^2 q_j) = \text{sgn}\ [q_i(M_{j,i}+M_{i,j})] \qquad (10)$$

The form of Eq. (10) is chosen to avoid division because of the possibility that q_i is zero.

The signs produced by Eqs. (9) and (10) are indefinite in certain important cases. Indefiniteness cannot be avoided but it can be rendered inconsequential, as will be discussed next.

In the case of a 180° rotation, $q_0 = 0$, $M_{k,j} - M_{j,k} = 0$, and numerical imprecision causes Eq. (9) to produce indefinite signs. Thus, if the signs of all three vector elements are determined by cyclically permuting the indices of Eq. (9) as proposed by Rupp and Hendley, any one of eight solutions can occur, only two of which are correct. Two, not one, are correct because for a 180° rotation the vector part of the quaternion can arbitrarily be reversed. The remaining six (incorrect) solutions are reflections of the rotation axis about the planes of the coordinate frame. If Eq. (9) could be processed without numerical error, then for 180° rotations all three vectors elements would be assigned positive signs, also generally incorrect.

Because Eq. (9) is indefinite only when the rotation angle is 180°, in which case the direction of the rotation vector can arbitrarily be reversed, Eq. (9) can *always* be used to determine the sign of *one* vector element, say q_i. Then, given q_i, Eq. (10) can be used twice to determine the two remaining signs, provided q_i is not zero. If q_i is zero, then $M_{j,i} + M_{i,j} = 0$ and Eq. (10) is indefinite.

It is necessary and sufficient that q_i be the vector element of largest magnitude in order for this combination of Eqs. (9) and (10) to produce correct signs in all cases.

Necessity is established by the case in which two vector elements are zero and the rotation angle is other than 180° so that the sign of the nonzero element is not arbitrary. Then if q_i is one of the zero elements, Eq. (10) may incorrectly determine the sign of the nonzero element. Now because of numerical imprecision, two elements both nominally zero may differ. For example, one may be 1×10^{-17}, the other 2×10^{-17} while the nonzero element is ½. Equation (10) may fail if q_i is either "zero" element and both are avoided only if q_i is the vector element of largest magnitude.

Sufficiency is established because if q_i is the vector element of largest magnitude then the signs of the remaining two elements are correctly assigned regardless of their magnitudes. If all three vector elements are zero, indefinite signs are assigned without consequence.

The algorithm just developed, valid for all cases, consists of the following. First compute the scalar element by Eq. (4) and the magnitudes of the vector elements by Eq. (6). Then find *i*, *j*, *k* in cyclic order such that

$$|q_i| \geq |q_j|; \qquad |q_i| \geq |q_k| \qquad (11)$$

Then

$$q_i = \text{sgn}\ (M_{k,j} - M_{j,k})\ |q_i| \qquad (12)$$

$$q_j = \text{sgn}\ (q_i(M_{j,i}+M_{i,j}))\ |q_j| \qquad (13)$$

and

$$q_k = \text{sgn}\ (q_i(M_{k,i}+M_{i,k}))\ |q_k| \qquad (14)$$

where

$$\text{sgn}\ (A) \triangleq \begin{cases} -1, & A<0 \\ +1, & A \geq 0 \end{cases} \qquad (15)$$

Equations (4) and (6) must be protected against square root failure because numerical imperfection can yield a negative argument which should be zero. Divide overflow can not occur because divisions by variables have been avoided.

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Lubricant Reservoir Systems: Thermal Considerations

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Nomenclature

A, B = constants for a liquid, Eq. (1)
H = molar heat of vaporization of liquid
M = molecular weight
p = pressure
R = universal gas constant
r = nylon pore radius
T = temperature, K
t = time
$\bar{V}$ = molar volume of liquid (molecular weight/density)
w = weight loss per unit area
γ = surface tension of liquid
ρ = density

Subscripts

p_0 = vapor pressure of bulk liquid at given temperature, Eq. (2)
T_B = chamber B temperature
T_C = chamber C temperature

Introduction

In an earlier paper,[1] it was determined that, under isothermal conditions, porous nylon blocks used as lubricant replenishment reservoirs in such spacecraft components as despun mechanical assemblies actually behave as lubricant sinks or sponges. The cause of such behavior is capillary effects. In this paper, the thermal conditions necessary to insure that a porous reservoir functions satisfactorily as a lubricant replenishment source are derived by thermodynamic methodology.

Several active feed types of reservoir systems have been proposed by spacecraft contractors. The primary problem is knowing the amount of lubricant to be released and the rate of its release. Computations of these quantities have been based only on the amount of lubricant lost by vapor phase trans-

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port, as predicted by the Langmuir evaporation rate equation

$$\mathrm{d}w/\mathrm{d}t = AT^{-1/2}\exp(-B/T) \tag{1}$$

Typically, no consideration has been given to surface transport (commonly called creep or migration). Yet, surface transport caused by capillary forces or temperature gradients can be orders of magnitude faster than vapor phase transport.[1] The only order-of-magnitude estimates of surface transport have been performed by the authors.[1,2] The neglect of capillarity and surface transport considerations is caused predominantly by the lack of adequate theoretical models and the difficulty in making reproducible measurements. In this paper, a simple model amenable to the thermodynamic analysis is described which circumvents these difficulties.

Theoretical Discussion

Because of the complexity inherent in describing the pressure gradients[1] that cause transport, only the conditions that control transport between a capillary system and bulk oil-covered surfaces, i.e., curved versus flat interfaces, are considered. The approach is essentially thermodynamic, and as such, the derived equations have the validity imparted to them by the first and second thermodynamic laws.

As a simple example of the model, consider placing a narrow-bore glass tube in a beaker of water. The water will rise in the tube until the capillary surface force equals the gravitational force. The model becomes useful when three points are recognized: 1) the capillary does not have to be in direct contact with the beaker as long as the system is closed (i.e., ultimately, the same equilibrium will be reached through the vapor phase); 2) the gravitational forces in spacecraft are essentially negligible; and 3) if the beaker and the tube are allowed to be at different temperatures, there should be temperatures at which no transport occurs. (The latter point becomes obvious when extreme temperatures are considered.)

The crux of the present approach is that, under isothermal conditions, liquids condense in capillaries at a lower pressure than they normally do. However, there are nonisothermal conditions, i.e., different temperatures, at which both capillary and bulk condensation will occur at the same pressure. These conditions are described quantitatively in the following discussion.

The problem is reduced to its simplest form in order that the thermodynamic free-energy equations can be written. The model chosen is shown in Fig. 1 The left and right chambers are both under isothermal conditions (but at different temperatures T_B and T_C, respectively) and are separated by a nonpermeable membrane that, when undistorted, indicates when pressure equilibrium is reached. Chamber B contains bulk liquid, and chamber C contains liquid in a capillary of radius r.

Pressure Balance Derivation

The well known Kelvin equation[3]

$$RT\,\ell n\, p/p_0 = 2\gamma\overline{V}/r \tag{2}$$

can be used as a basis for establishing a pressure balance between the two chambers. p is the pressure at which vapor condenses in a capillary of a given radius at a given temperature and is denoted by p_C in the right-hand chamber of Fig. 1. At pressure equilibrium, the pressure in both chambers must be equal, i.e., $p_B = p_C$. Then, Eq. (2) becomes

$$\ell n\, p_B = \ell n\, p_C = \ell n\, p_0 + (2\gamma\overline{V}/rRT_C) \tag{3}$$

Further simplification may be made because, in each chamber, the conditions are such that there is only one degree-of-freedom. Thus, temperature rather than pressure may be used as the independent variable, and the equation recast by the use of the approximate equation of state

$$\ell n\, p = A + B/T \tag{4}$$

where A and B are experimental system constants usually determined utilizing the Langmuir evaporation equation. Substitution of Eq. (4) for both p_B and p_0 in Eq. (3) yields

$$A + B/T_B = A + B/T_C + 2\gamma\overline{V}/rRT_C \tag{5a}$$

or

$$(T_C - T_B)/T_B = \Delta T/T_B = 2\gamma\overline{V}/rBR) \tag{5b}$$

which is the desired relationship between T_B and T_C to insure that no net transport occurs between the chambers.

Because B is related to the enthalpy function H by $B = -H/R$,

$$\Delta T/T_B = -2\gamma\overline{V}/rH \tag{6}$$

As r approaches infinity, i.e., a flat surface, ΔT approaches zero. The equation does not hold for very small r (<100 Å) because surface effects are then comparable to volume effects. At such small radii, γ, $\overline{V}$, and even H become functions of r.

Numerical Example for Apiezon C

In order to evaluate Eqs. (5) for Apiezon C, they can be rewritten as

$$\Delta T/T_B = 2\gamma M/r\rho BR = J/r \tag{7}$$

The largest uncertainty occurs in the constants M and B. This uncertainty is a reflection of the basic inhomogeneity of the lubricant. Because the range of pore diameters in porous nylon reservoirs extends over at least an order of magnitude (0.2-2 μm),[4] more accurate determinations of molecular weight are not presently required.

The values used in the equation were: $\gamma = 32$ dynes-cm^{-1}, $M = 540$ g-mole^{-1}, $\rho = 0.874$ g-cm^{-3}, $B = 11,800$ deg^{-1}, $R = 8.314 \times 10^7$ erg-mole-deg^{-1}, and $J = 2\gamma M/\rho BR = 4.0 \times 10^{-8}$ cm^{-1} $= 4.0 \times 10^{-4}$ μm^{-1}. In most satellite experiments, temperatures were within 20° C of room temperature. Thus, further simplification can be obtained by using $T_B = 300$ K and

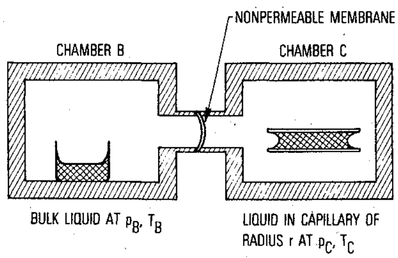

Fig. 1 Experiment: idealized lubricant in bulk (*B*) state vs lubricant in capillary (*C*) state. Problem: when $p_B = p_C$, find T_B for any T_C.

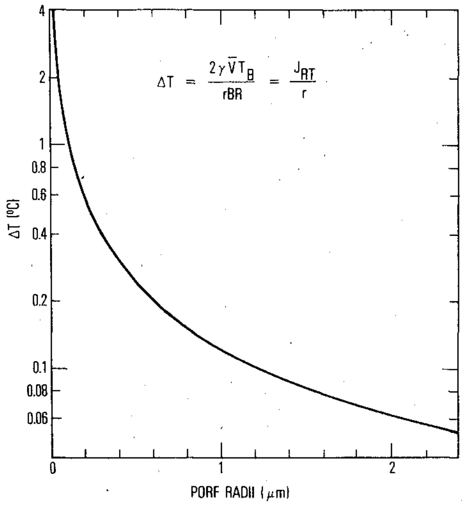

Fig. 2 Temperature differential required for no net transport between a bulk liquid reservoir and a porous liquid reservoir.

defining J_{RT}

$$\Delta T = JT_B/r = J_{RT}/r = 0.12/r \qquad (8)$$

where ΔT is in degrees celsius and r is in micrometers. For typical pore diameters of from 0.2 to 2 μm, therefore, the temperature differences required to maintain lubricant transport from reservoirs to their surroundings are from 0.6 to 0.06° C. Figure 2 is a graph of Eq. (7) for T = 300 K (26.8°C, 80.3°F).

Discussion

The lubricant transfer rate is governed largely by the geometry between the bearing and the reservoir and by the fashion in which the pores of various geometries are connected. The validity of the discussion is limited by the neglect of these parameters.

The important consequence of Eq. (7) is that it is first quantitative estimate of the temperature gradient necessary for a reservoir to function properly. The available temperature information, from satellites where such reservoir systems are used, indicates that some of the reservoirs are colder than the bearing area for a significant (if not major) part of the time. For such systems, one may justifiably question their utility.

If it is assumed that, to achieve reasonable transfer rates, the ΔT will be within a factor of 2 to 5 times that predicted by Eq. (7), i,e., about 2 to 4° F, further conclusions can be drawn. This temperature gradient is sufficiently small that the naturally occurring temperature gradients within the satellite could insure proper function. For such a configuration, it should be useful to construct reservoirs of high thermal conductivity materials.

In order to achieve a reasonable degree of predictability, the main source of variability, the pore size distribution, should be better defined. The only limitation is that the reservoir should not empty under the forces incurred during launch.

In summary, we have shown that the temperature of the reservoirs must be greater than that of the surroundings, but that the necessary temperature gradient can be quite small. The results also indicate that the most suitable reservoir material should have a high thermal conductivity and a near-homogeneous pore size distribution.

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Effect of Mass Asymmetry on Ballistic Match of Projectiles

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Nomenclature

cg = center of gravity
C_A = axial force coefficient
C_{m_0} = aerodynamic asymmetry induced pitching moment coefficient
$C_{M_{p\alpha}}$ = symmetrical Magnus moment slope coefficient (based on $d/2U$), 1/rad
$C_{M_q} + C_{M_{\dot\alpha}}$ = symmetrical pitch damping derivative coefficients (based on $d/2U$), 1/rad
C_{M_α} = symmetrical pitching moment slope coefficient ($C_{M_\alpha} > 0$ for shell), 1/rad
C_{n_0} = aerodynamic asymmetry induced yawing moment coefficient
C_{N_α} = symmetrical normal force slope coefficient, 1/rad
d = body diameter (reference length), ft or m
i = $(-1)^{1/2}$
I, I_X = lateral and axial moments of inertia, respectively, slug-ft^2 or kg-m^2
J_{XY}, J_{XZ} = products of inertia, slug-ft^2 or kg-m^2
m = vehicle mass, slug or kg
p = roll rate, rad/sec
q' = dynamic pressure, lb/ft^2 or N/m^2
S = reference area, ft^2 or m^2
S_g = gyroscopic stability factor ($S_g > 1$ necessary for stable flight)
t = time, sec
U = total velocity, fps
y_{cg}, z_{cg} = body fixed lateral position of cg relative to origin of reference coordinate system, ft or m
α, β = nonrolling angles of attack and side-slip, respectively, rad or deg
ξ = complex total angle of attack relative to nonrolling coordinates, rad or deg

Subscripts

fm = first maximum yaw
0 = conditions at muzzle exit ($t = 0$)
1,2,3,4 = nutation, precession, trim, and yaw of repose, respectively

Introduction

To ballistically match similar artillery shells for identical muzzle exit conditions (i.e., make their mean impacts fall within some specified small range and deflection probable errors), it was shown by Vaughn and Wilson[1] that it is important to match the ballistic coefficients and yaw of repose induced drifts of the projectiles. Results presented herein indicate that in addition to these requirements it is necessary to control, within some permissible upper bound, the magnitude of the projectiles' initial angular motion. These initial disturbances can alter the flight path to produce trajectory deflections[2] and, through angle-of-attack induced drag, produce changes in ballistic coefficient. Both effects can result in unacceptably large relative range and/or deflection errors. The initial disturbances fall into two categories: 1) those introduced by the gun, which are usually small, and 2) those which result from projectile asymmetries. The latter category is investigated in this Note.

Small mass asymmetries (principal axis misalignment and lateral cg offset) inherent in certain artillery shells can have large effects on the magnitude of the projectiles' initial angular motion. It is shown herein that even when the fundamental ballistic similitude criterion is satisfied (i.e., the shapes, weights, moments of inertia, and static margins are matched) and identical muzzle exit conditions are achieved, a poor ballistic match can result when small differences in the level of principal axis misalignment and lateral cg offset exist between projectiles. The cg offset effect referred to here, which is indistinguishable from that of principal axis misalignment, is a gun twist dependent contribution to trim angle that is important for major caliber projectiles. This contribution, which has often been ignored in the past, is not to be confused with the better known cg offset induced

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